$\beta_c a_1$ must be kept at the mean frequency of the operating band, when wide-band couplers are needed. In practical cases the mean frequency and the dimensions of the inner helix are given, and the value of h is determined by the characteristic impedance of the connecting cable. Thus βa_1 and h/a_1 are known, and from these values the ratio of the two radii have to be determined to get the maximum at a given value of βa_1.

In order to determine the position of the maximum, β_c must be differentiated with respect to β and the differential quotient made equal to zero. And so the equation

$$\frac{a_2}{a_1} - 1 - \frac{1}{\beta a_1} + \left(\frac{a_2}{a_1} - 1 + \frac{h}{a_1}\right) \cdot \frac{e^{-2\beta a_1((a_2/a_1)-1+(h/a_1))}}{1 - e^{-2\beta a_1((a_2/a_1)-1+(h/a_1))}} - \frac{h}{a_1}\,\frac{e^{-2\beta a_1(h/a_1)}}{1 - e^{-2\beta a_1(h/a_1)}} = 0 \qquad (8)$$

is obtained. In the limit case, when $h/a_1 = \infty$, (8) has the simpler form

$$\frac{a_2}{a_1} - 1 - \frac{1}{\beta a_1} = 0. \qquad (9)$$

This corresponds to the case when the distance of the shield to the coupling helix is infinite, that is when the coupling helix is not shielded. Consequently, the effect of the shield is given by the fourth and fifth terms at the left side of (8).

The solution of (8) cannot be given in an explicit form. Therefore, this equation has been solved numerically for several values of βa_1 and h/a_1 by successive approximations. In Fig. 2 the optimum values for the ratio of two radii a_2/a_1 obtained by solving (8) are given as a function of βa_1 for different values of h/a_1. It is apparent that, taking the effect of shielding into account, a higher value for the optimum ratio of radii is obtained than without the shield. Moreover, it is to be seen that decreasing the spacing between the coupling helix and the shield, the optimum ratio of the radii is increased.

T. Berceli
Telecommun. Research Institute
Budapest, Hungary

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Dispersion of Pulsed Electromagnetic Waves in a Plasma

Observations of the dispersion of electromagnetic pulses in isotropic plasmas [1] and gyrotropic plasmas [2] show that the response is distorted due to the excitation of weakly damped oscillations characteristic of the natural frequencies of the plasma. For diagnostic application, signals with a smooth frequency spectrum, such as steps or short unidirectional pulses, allow a simple correlation between the observed response and the relevant plasma parameters. In radar or communications systems, however, the dispersion of sinusoidal pulses carrying information in the pulse amplitude or in the pulse duration is important. Due to the relatively high spectral intensity near the carrier frequency, a pronounced signal distortion arises if the carrier frequency is comparable to a natural oscillation frequency of the plasma.

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We have calculated the distortion of a pulsed electromagnetic wave (angular frequency ω_s) caused by a reflection from a semi-infinite uniform and isotropic plasma (angular plasma frequency ω_p), and the distortion of a pulsed wave transmitted through a plasma by convolution of the response for a short unidirectional pulse. For the reflected wave, the impulse response is [1]

$$v_r(t) = -\frac{2J_2(\omega_p t)}{t}. \qquad (1)$$

Hence, for a carrier pulse of duration T_0 and amplitude V_0,

$$V_r(t) = V(t) - V(t - T_0)$$

with

$$V(t) = -2V_0 \int_0^t \sin\omega_s(t-\tau)\,\frac{J_2(\omega_p\tau)}{\tau}\,d(\tau). \qquad (2)$$

Computer results for three different ratios between the carrier frequency and the plasma frequency are shown in Fig. 1(a) as a function of time. For comparison with the experimental observations, a pulse duration of three cycles has been assumed. The initial response is dominated by low-frequency components of the incident carrier pulse $\omega < \omega_p$, for which the magnitude of the reflection coefficient is large ($r = -1$). The first peak is inverted with respect to the polarity of the incident wave and is progressively delayed in time for lower electron densities. The response subsequently tends to approach the steady-state condition for the assumed carrier frequency. Following the termination of the pulse, a characteristic transient ringing arises, which in time approaches the plasma frequency. The amplitude of this oscillation simply reflects the spectral intensity of the exciting signal near the plasma frequency.

A short voltage impulse set up inside an unbounded plasma at $z = 0$ gives rise to a signal at $z = d$ of the form [1]

$$v_T(t) = \delta(t - d/c_0) - \omega_p \frac{d}{c_0}\,\frac{J_1(\omega_p\sqrt{t^2 - (d/c_0)^2})}{\sqrt{t^2 - (d/c_0)^2}} \qquad (t \geqq d/c_0). \qquad (3)$$

Hence, the transmitted signal response for a carrier pulse of duration T_0 is

$$V_T(t) = V(t) - V(t - T_0)$$

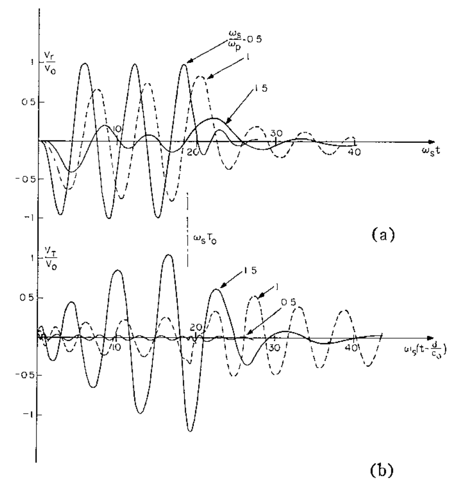


Fig. 1. Dispersion of short sinusoidal pulses in a plasma. Pulse duration 3 cycles, angular carrier frequency ω_s, amplitude V_0 (a) pulse reflected from a plasma-air interface. (b) pulse transmitted through a distance d in a plasma ($\theta = 15$).

with

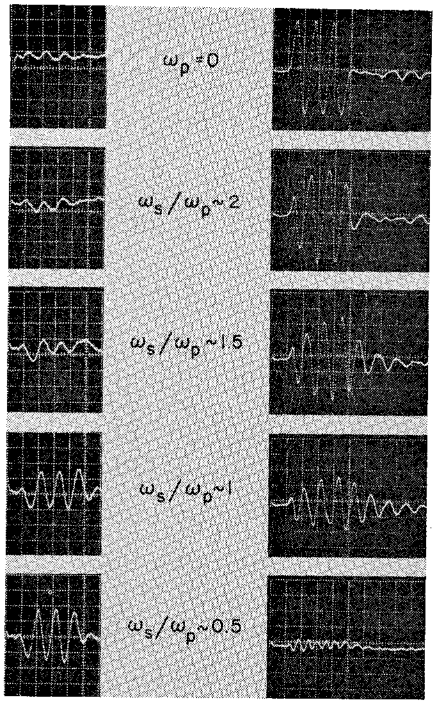
$$V(t) = V_0\left\{\sin(\omega_s t - \theta) - \frac{\theta}{\omega_s/\omega_p}\int_{\omega_s\tau=\theta}^{(\omega_s t)} \sin\omega_s(t-\tau) \cdot \frac{J_1((\omega_p/\omega_s)\sqrt{(\omega_s\tau)^2 - \theta^2})}{\sqrt{(\omega_s\tau)^2 - \theta^2}}\,d(\omega_s\tau)\right\} \qquad (\omega_s t \geqq \theta), \qquad (4)$$

where $\theta = \omega_s d/c_0$.

The response shown in Fig. 1(b) for $\theta = 15$ exhibits a slow build-up of the RF oscillation in underdense plasmas ($\omega_s/\omega_p = 1.5$ and 1.10), followed by a persistent transient oscillation near the plasma frequency after termination of the incident signal. The build-up time of the initial response and the persistence of the trailing oscillations are longest for a plasma with a critical electron density $\omega_p = \omega_s$, and increase with increasing distance from the source [3]. In an overdense plasma ($\omega_s/\omega_p = 0.5$), the main signal is cut off. Instead, weakly damped oscillations near the electron plasma frequency are excited at the beginning and at the end of the carrier pulse.

The excitation of the predicted transient oscillations and their decay in time have been observed experimentally by measuring the dispersion of short wave trains at a frequency of about 500 Mc/s in a neon afterglow plasma, confined in a coaxial transmission line 1.5 m long ($\theta \sim 15$). The apparatus is more fully described in Schmitt [1] and [2]. For the purposes of the present experiment, a phase-locked sinusoidal pulse with three cycles is generated by feeding a single three separate transmission lines. By adjust-pulse with equal rise and fall times into three separate transmission lines. By adjusting the delay of each of the resultant pulses and recombining them in a single transmission line, a series of equally spaced pulses is obtained. The short RF pulse is generated by differentiating this pulse sequence by means of a small-series capacitor (approximately 2 pF in a 50-ohm transmission-line system).

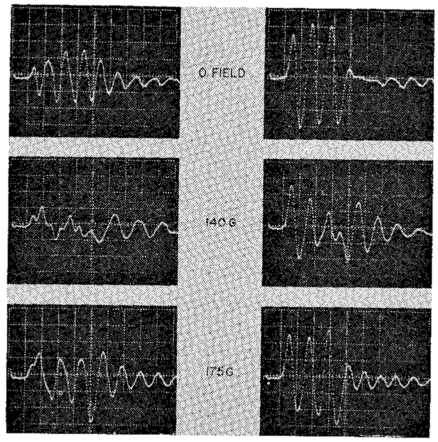
In the absence of ionization, the coaxial line is approximately matched to the con-



(a) (b)

Fig. 2. Observed dispersion of sinusoidal pulses in a plasma-filled coaxial line. Time scale 2 ns per division, pulse duration 3 cycles. (a) pulse reflected from plasma-air interface. (b) pulse transmitted through plasma ($d = 1.5$ *m*, $\theta \sim 15$).

necting transmission line. The reflection of the wavetrain at the interface between the coaxial line and the connecting cable is shown in Fig. 2(a) ($\omega_p = 0$). The RF pulse proceeds essentially undistorted through the coaxial line, where it is detected by a small field probe [Fig. 2(b) ($\omega_p = 0$)]. A spurious response, delayed by approximately 12 ns, is caused by rereflection of the main pulse after its repassing twice through the coaxial line. The enhanced reflection of the pulse and the distortion of the transmitted pulse caused by the plasma are shown in the following exposures [Figs. 2(a) and 2(b)], observed at various times in the afterglow corresponding to different values of the plasma frequency. In an underdense plasma ($\omega_s/\omega_p = 2$ and 1.5), the reflection remains small. The reflected signal, and particularly the transmitted signal, show the development of a pronounced transient ringing near the plasma frequency. The duration of the observed transient is longest for a plasma with near-critical electron density ($\omega_s/\omega_p \sim 1$). Clearly, the received signal bears little resemblance to the original short-pulse train. In an overdense plasma ($\omega_s/\omega_p \sim 0.5$), the reflection is large as the main pulse is cut off in the plasma. The small transmission observed arises from frequency components of the pulse spectrum near and beyond the plasma frequency, enhanced by spurious components near the harmonic of the carrier frequency due to imperfect sinusoidal-pulse shape. In a thin plasma layer, a weak tunneling of the attenuated main pulse is also observed [4]. For a comparison with the calculated results, note that for the transmitted signal, interface reflections which tend to reduce the transient duration have



(a) (b)

Fig. 3. Dispersion of a sinusoidal pulse in a longitudinally-magnetized plasma in a coaxial line. Time scale 2 ns per division, magnetic field in gauss. (a) $\omega_s/\omega_p \sim 1$. (b) $\omega_s/\omega_p \gg 1$.

been neglected in the theoretical response.

The complexity of the dispersion process is increased if static magnetic fields are superimposed on the plasma. For a longitudinally magnetized plasma in a coaxial line, one obtains a dispersion relation [5]

$$k^2 = \frac{\omega^2}{c_0^2}\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}\right) \qquad (5)$$

which reflects pass bands below the angular electron-cyclotron frequency ω_c and above the hybrid frequency $\omega_h = \sqrt{\omega_p^2 + \omega_c^2}$. The distortion of a wave pulse will depend on the relation between its carrier frequency and the natural frequency of the plasma.

We have observed experimentally the gross effects on the pulse dispersion introduced by a magnetic field set up by a long solenoid surrounding the plasma-filled coaxial line. The results, as shown in Fig. 3(a), indicate that for a plasma with near-critical electron density, a weak magnetic field tends to suppress the main pulse (Fig. 3(a), 140 gauss) as the upper cutoff frequency is shifted from ω_p to ω_h. The pulse is distorted into a weakly damped ringing, approaching in time the cyclotron frequency and arising from frequency components in the lower pass band $\omega < \omega_c$. With further increasing magnetic field, the main pulse is again observed when the lower pass band extends to frequencies $\omega_c > \omega_s$. The distortion vanishes for $\omega_c \gg \omega_s$, where transverse motion of electrons is inhibited by the strong magnetic field. The characteristic ringing at the electron-cyclotron frequency is particularly well observed in a tenuous plasma (Fig. 3(b)) where negligible distortion of the transmitted pulse occurs in the absence of magnetic fields. A nearly undamped oscillation at the cyclotron frequency trails the direct signal (Fig. 3(b), 175 gauss). Similar transient oscillations are also observed in the reflected pulse shape.

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H. J. Schmitt
Sperry Rand Research Ctr.
Sudbury, Mass.

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Injection Locking of a Laddertron at 35 Gc/s

An OKI 34LV10 Laddertron was locked at 34 830 Mc/s by injecting a controlling signal into the Laddertron cavity. The Laddertron is a single-cavity multigap oscillator that can deliver about 10 watts CW. The controlling signal was obtained from a klystron that was phase locked to a crystal oscillator harmonic. The Laddertron can be locked for CW or for pulse operation.

Phase locking of microwave oscillators is usually accomplished by using an intermediate frequency offset. The principal advantage of this method is that the reference power required is only what is needed to operate a limiter in the IF amplifier. The disadvantages are a) electronic circuits are required, b) a microwave directional coupler and mixer are required, and c) the spectrum of the output is that of the reference input only within the pass band of the lock loop (typically 100 kc) so that the electronic loop determines the modulation capability. Therefore, frequency control is not possible for pulse operation.

The injection-locked system requires no electronic circuitry and the modulation capability is determined by the response time of the locked oscillator. The only microwave component needed is a circulator. The main disadvantage is that more reference power is required; the reference power for the IF phase lock can be obtained from a maser oscillator (10^{-10} watts) or from a harmonic of a crystal driven by an RF source while the reference power for the injection lock is typically 30 dB below the oscillator output.

Adler's theory [1] of oscillator synchronization was verified for *X*-band reflex klystrons by Mackey [2]. The locking relationship is

$$\left[\frac{2Q\Delta f}{f_0}\left(\frac{P_0}{P_1}\right)^{1/2}\right] = \sin\phi \qquad (1)$$

where

P_0 = oscillator output power
P_1 = injected input power
$\Delta f = f_0 - f_1$
f_0 = free-running oscillator frequency
f_1 = injected input frequency

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